Calculating Dimensionless Physical Constants

by Dr. Bruce D. McLaughlin

A dimensionless physical constant is an essential numerical characteristic of space, time, matter and energy expressed as a pure number. Such constants play a central role in physics because they are intrinsically invariant under changes in units of measurement. Historically, these constants were foundational because they could not be derived from fundamental principles. However, in the 1920's, Arthur Eddington pursued the idea that dimensionless physical constants might be obtainable from pure mathematics. The workings of the universe might emerge solely from abstract, unit-free mathematics without need of empirical input, beyond establishing a starting point for calculations. Eddington's efforts were generally not successful. This technical note is intended to revive his efforts and take them to the next level. Two dimensionless physical constants are used for illustration: (1) the Fine Structure constant (α) and (2) the Electron/Proton rest mass ratio (γ). If the proposed conjecture is theoretically sound, then Arthur Eddington has been vindicated on this matter.

Analysis

The fundamental conjecture is that at least one infinite power series exists in the variable ω (dimensionless physical constant), where $1>\omega>0$, of the form

$$\Delta(\omega) = \sum_{n=1}^{\infty} F_n(\tau, \varphi, \omega) = 0 \tag{1}$$

This power series converges to zero and satisfies the Ratio Test for convergence. τ and φ represent two of the four numbers π, 22-6$\sqrt{13}$, 2$\sqrt{3}$ and 22+6$\sqrt{13}$. The selection of these particular numbers will be addressed later.

The first step in specifying $\Delta(\omega)$ is to select an estimate for ω; the purpose of this estimate is to allow selection of exponents in successive terms of the infinite series. The number 0.0072973525643 was used as an estimate for the Fine Structure Constant, and the number 0.000544617 was used as an estimate for the Electron/Proton rest mass ratio. If subsequent calculated values of ω are accurate to more digits than the estimates, that feature cannot be attributed to the estimates alone.

The first term in the series is $-\omega^{-1}$. The second term is $\tau^x\varphi^y$ where x and y are selected to make $-\omega^{-1}+\tau^x\varphi^y$ as close to zero as possible. The third term is a simple function of τ and φ multiplied by ω to make $-\omega^{-1}+\tau^x\varphi^y+f(\tau,\varphi)\omega$ as small as possible. Each remaining term is of the form $\tau^\theta\varphi^\vartheta\omega^\mu$ in which μ is a positive integer increasing by 1 in each successive term. If

$\tau>1$ then θ is a positive integer; if $\tau<1$ then θ is a negative integer. If $\varphi>1$ then ϑ is a positive integer; if $\varphi<1$ then ϑ is a negative integer. By choice, $\tau>\varphi$.

The estimation of ω^{μ} is used as a starting point for each term after the third. Then $|\theta|$ is increased until $\tau^{\theta+1}\omega^{\mu}(\vee\tau^{\theta-1}\omega^{\mu}$ if θ is negative) is *barely greater* than the magnitude of the previous sum. In other words, $\tau^{\theta}\omega^{\mu}$ *is barely less than the magnitude of the previous* $\sum$. Next, $|\vartheta|$ is increased until $\tau^{\theta}\varphi^{\vartheta}\omega^{\mu}$ is *barely less* than the magnitude of the previous sum. In other words, $\tau^{\theta}\varphi^{\vartheta+1}\omega^{\mu}$ is *barely greater* than the magnitude of the previous sum. Finally, a sign is affixed to the term to ensure that the magnitude of the next sum is smaller than the magnitude of its predecessor. This rule is unambiguous and produces a unique term for each index which makes the series mathematically legitimate. This rule also forces the series to satisfy the Ratio Test and converge to zero.

After each term is added, the sum of terms is set equal to zero and NSolve or FindRoot (Mathematica) can be employed to solve for ω which is viewed as the only unknown in the equation. This step in the process is weakened by two primary factors. First, the precision limitations of Mathematica require truncating and restarting the calculation about every 30 terms. Second, the infinite series is truncated at a particular term for every calculation; a partial sum and not the infinite sum is set equal to zero. The overall influence on precision, of these two shortcomings, has not been determined.

If a bad choice is made for the exponents in a particular term and the magnitude of the partial sum is increased, then one to three more iterations will be required to make a correction and lock in a digit. One to three iterations locks-in a digit in the expression for ω and that digit will not change with further iterations. This entire manual process could be automated by an appropriate program.

The overall approach, to computing dimensionless physical constants, represents a self-consistent, nearly fixed-point model where a constant is defined implicitly by a series whose terms depend on the constant itself. Furthermore, the last computed term in the series is virtually the same order of magnitude as the last computed sum of terms. If dimensionless physical constants have their origin in relatively simple infinite series and not in the deep recesses of quantum mechanics, string theory and relativity, then mathematics is truly the cornerstone of science.

Fine Structure Constant

For the Fine Structure constant, Equation (1) can be written as:

$$\Delta(\alpha)=\sum_{n=1}^{\infty}F_n(b,\pi,\alpha)=0 \qquad (2)$$

which converges to zero and satisfies the ratio test for convergence. The variables b = $22+6\sqrt{13}$ and π = 3.141592...are independent parameters and πb = 137.078… which is nearly equal to α^{-1} = 137.035… The series structure used for computation is:

$$\Delta(\alpha) = -\alpha^{-1} + b\pi - (2\pi - 1/2)\alpha + 2\alpha^2 + b\alpha^3 - b\pi^2\alpha^4 + b^2\pi\alpha^5 \ldots \tag{3}$$

Each remaining term in the series is of the form $b^{\theta}\pi^{\vartheta}\alpha^{\mu}$ and each successive term increases the value of μ by 1. The final term in the current trial is $-b^{64}\pi^2\alpha^{78}$ which corresponds to $-1.860*10^{-61}$ while the sum of terms is $-6.988*10^{-62}$. After each term is added, the algebraic expression for sum of terms is set equal to zero and NSolve or FindRoot (Mathematica) is used to solve for α.

The calculated value of α with a quasi-precision of 60 is:

0.0072973525642999992416860274723885002506940170167664094552769

compared to the current CODATA 2022 value of:

0.0072973525643

But keep in mind the unresolved precision issues associated with this approach.

If directionally correct, this functional relationship between b, π , and α shows the promise of allowing the Fine Structure Constant to be computed with arbitrary precision. It would also address the concerns of Feynman on this issue:

> Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by humans. You might say the "hand of God" wrote that number, and "we don't know how He pushed His pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out – without putting it in secretly! *Feynman, R. P. (1985). QED: The Strange Theory of Light and Matter. Princeton University Press. p. 129. ISBN 978-0-691-08388-9*

Electron/Proton Rest Mass Ratio

For the Electron/Proton Rest Mass Ratio, Equation (1) can be written as:

$$\Delta(\gamma) = \sum_{n=1}^{\infty} F_n(b, c, \gamma) = 0 \qquad (4)$$

which converges to zero and satisfies the ratio test for convergence. The variables b = 22+6$\sqrt{13}$ and c = 2$\sqrt{3}$ are independent parameters and bc^3 = 1813.79… which is nearly equal to γ^{-1}= 1836.15267… The series structure used for computation is:

$$\Delta(\gamma) = -\gamma^{-1} + bc^3 + (1/2)b^2c^3\gamma + b^3c^3\gamma^2 - b^4c^5\gamma^3 + b^5c^6\gamma^4 + b^8c^3\gamma^5 \ldots \qquad (5)$$

Each remaining term in the series is of the form $b^{\theta}c^{\vartheta}\gamma^{\mu}$ and each successive term increases the value of μ by 1. The final term in the current trial is $-b^{156}\gamma^{96}$ which corresponds to -2.995*10^{-58} while the sum of terms is -7.998*10^{-59}. After each term is added, the algebraic expression for sum of terms is set equal to zero and NSolve or FindRoot (Mathematica) is used to solve for γ.

The calculated value of γ with a precision of 60 is:

0.000544616999999999920804463029216992919609032592001902690208433

compared to the current value of:

0.000544617021

But keep in mind the unresolved precision issues associated with this approach.

Source of the Parameters

Although the author had a particular reason for selecting the numbers π, 22-6$\sqrt{13}$, 2$\sqrt{3}$ and 22+6$\sqrt{13}$ as potential values for $\tau$and φ, the reader does not have to accept the legitimacy of this reason to accept the legitimacy of the algorithm presented herein. In other words, the reader can view these four numbers as serendipitous guesses without affecting the algorithm for determining dimensionless physical constants. **However, in truth, these four numbers were extracted from the first nine characters of Genesis in the Hebrew Bible.**

Let us begin with π . First invert the character sequence, in the first chapter of Genesis, so the characters are read from left to right. Next connect the first three contiguous characters in the first verse of Genesis to form a string. Now represent each of the Hebrew characters by a base-3 triplet (column vector) according to a rule that starts with Aleph as $(000)^T$ and ends with Tsadey Final as $(222)^T$. The three column vectors are $(001^T,201^T,000^T)$. These three base-3 triplets create a 3 by 3 matrix having the elements 0, 1 and 2:

$$Ayz = \begin{matrix} 020 \\ 000 \\ 110 \end{matrix} \qquad (6)$$

The first eigenvector of $(Ayz)(Ayz)^T$ is $(\rho,0,1)$ where ρ is the Golden Ratio which is, in turn, related to π by:

$$\pi = (5) \cos^{-1}(\rho/2) \qquad (7)$$

The Golden Ratio and therefore π are therefore concealed in the first three Hebrew characters of Genesis.

A path to the remaining three numbers is a bit more tedious. First, introduce the column vectors $(202^T,100^T,210^T)$ and $(001^T,201^T,000^T)$ representing the second three and third three characters in the first verse of Genesis. These base three triplets create two additional 3 by 3 matrices:

$$Byz = \begin{matrix} 212 \\ 001 \\ 200 \end{matrix} \qquad (8)$$

$$Cyz = \begin{matrix} 020 \\ 000 \\ 110 \end{matrix} \qquad (9)$$

Note that Ayz and Cyz are identical, so we really have only two independent matrices with 0, 1 and 2 as elements. Now stack these three matrices together to form what looks like a Rubic Cube. Each of 15 slices through the cube produces a 3 by 3 array; three slices each parallel to the yz, zx and xy planes and two diagonal slices perpendicular to each plane (denoted by X).

Each slice can be rotated about various axes to produce eight 3 by 3 matrices: dihedral group of order eight (D_4). In this trial, only one matrix is selected for each slice. These 15 matrices can then be combined by matrix multiplication to form

$$\begin{aligned} yz &= Ayz.Byz.Cyz.Xyz.Xzy \\ zx &= Azx.Bzx.Czx.Xzx.Xxz \qquad (10) \\ xy &= Axy.Bxy.Cxy.Xxy.Xyx \end{aligned}$$

Finally, to create symmetry,

$$\begin{aligned} x &= yz.yz^T \\ y &= zx.zx^T \qquad (11) \\ z &= xy.xt^T \end{aligned}$$

Now use the instructions in the publication "Characteristic polynomial and higher order traces of third order three dimensional tensors," by Guimei Zhang and Shenglong Hu, Front. Math China 2019, 14(1): 225-237 to create a characteristic equation of degree 12. The non-zero solutions to this equation are $22-6\sqrt{13}$, $22+6\sqrt{13}$, $-2+2i\sqrt{3}$, and $-2-2i\sqrt{3}$.

By pure conjecture, the numbers $22-6\sqrt{13}$, $22+6\sqrt{13}$, $2\sqrt{3}$ and π were selected to create various infinite series that satisfied the Ratio Test and converged to zero.

What are the implications of all these findings? Do they represent a completely serendipitous concurrence of physics, mathematics and Hebrew text? Or did our transcendent, immanent, infinite, eternal and immutable God provide us with a user manual encrypted within His inspired, inerrant and infallible *Word*. The first verse of Genesis says: "In the beginning God created the essence of the heavens and the essence of the earth." What a perfect place to encrypt a user manual revealing the physics and mathematics of this new *creation from nothing* (bara). But why would God bother to encrypt a message within a message? A partial answer might be found in the New Testament.

> "For since the creation of the world, God's invisible qualities – His eternal power and divine nature – have been clearly seen, *being understood from what has been made*, so that men are without excuse" (Romans 1:20).

Only the first nine characters of Genesis have been used in this article. The concatenated words of the Torah comprise a 304,805 string of Hebrew characters. What mysteries might be encrypted therein?